Small area estimation of zero-inflated, spatially correlated forest variables using copula models

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Vicente Monleon, Lisa Madsen and Lisa Wilson

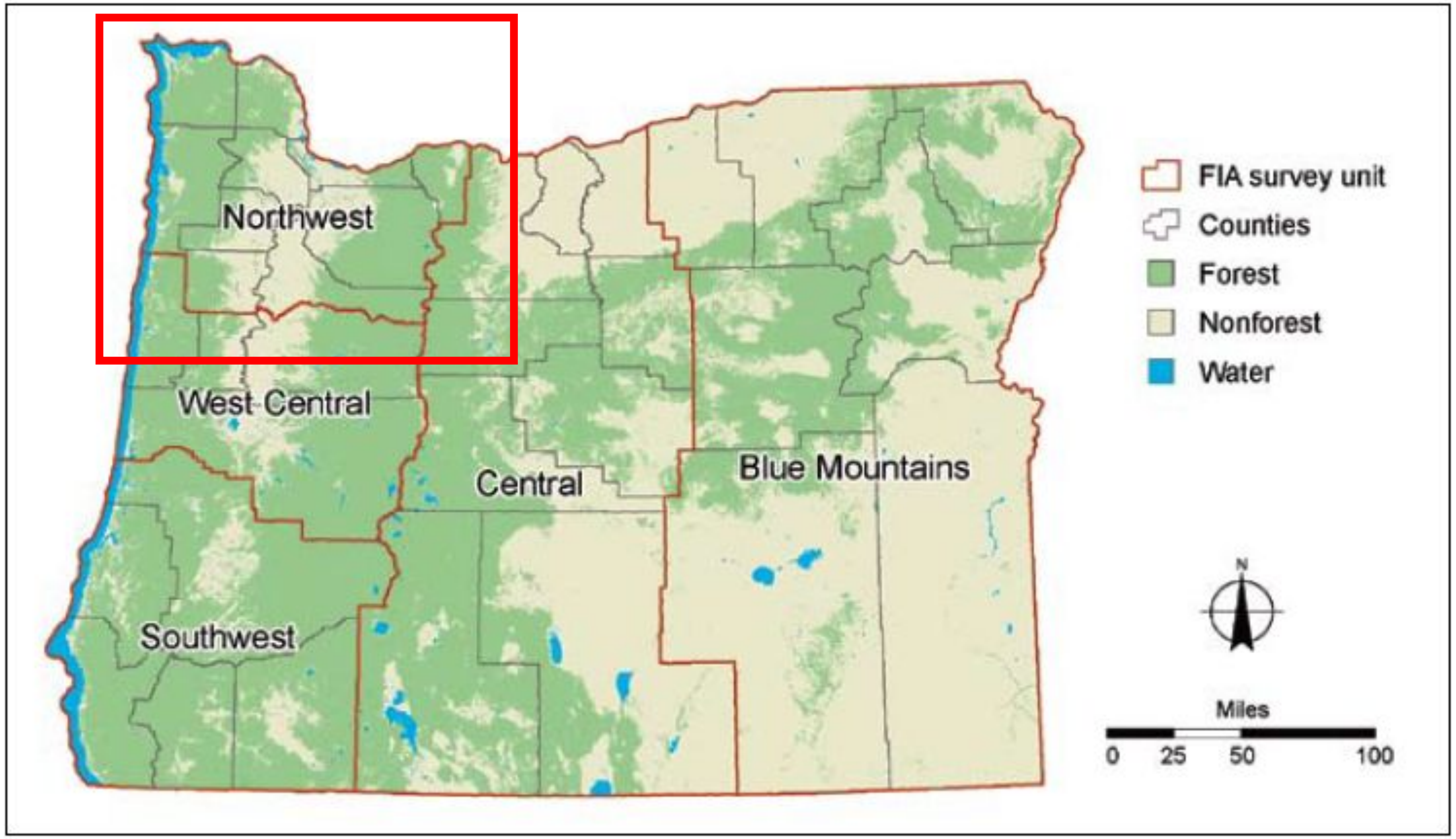
USDA Forest Service, PNW Research Station

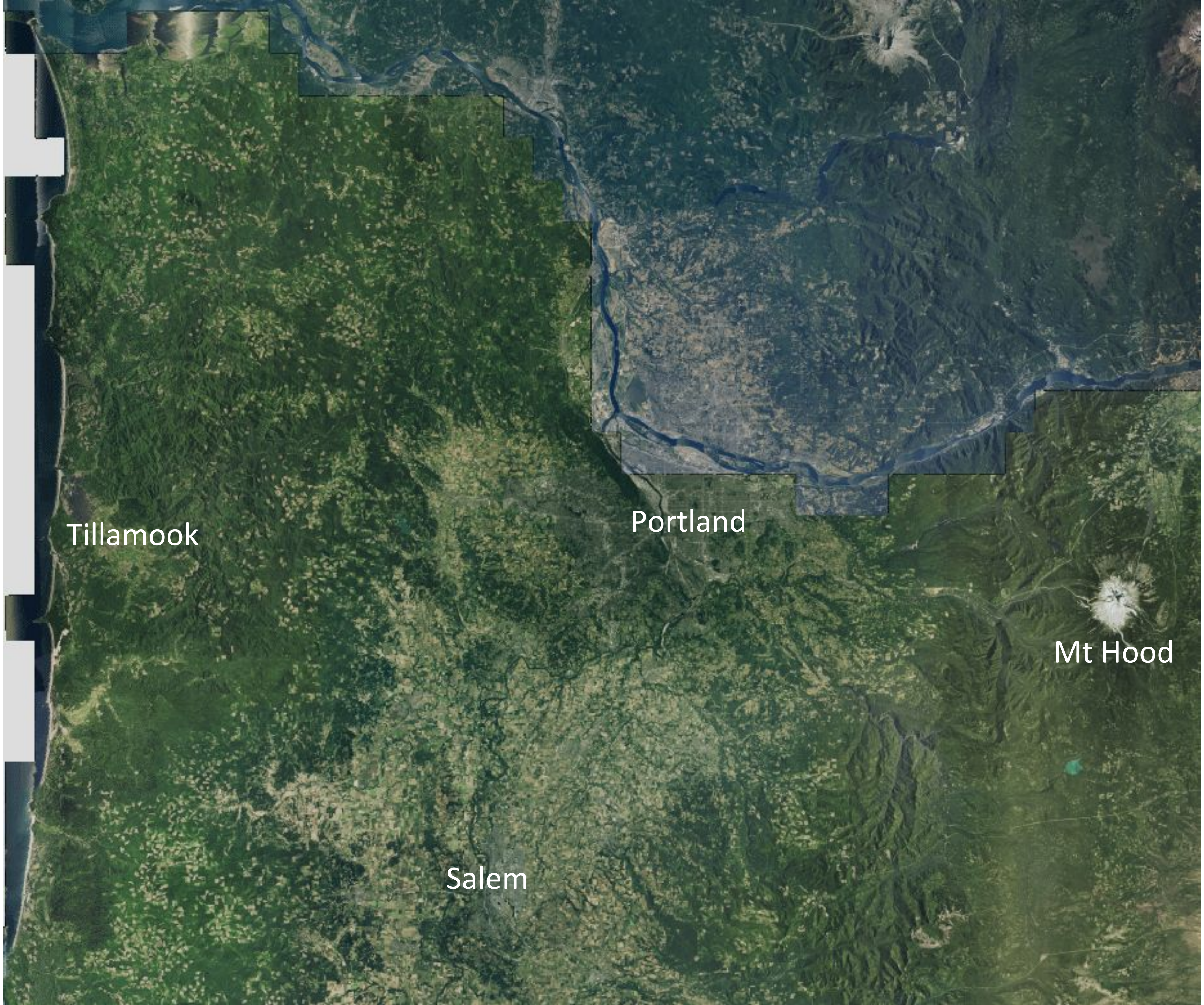
Department of Statistics, Oregon State University

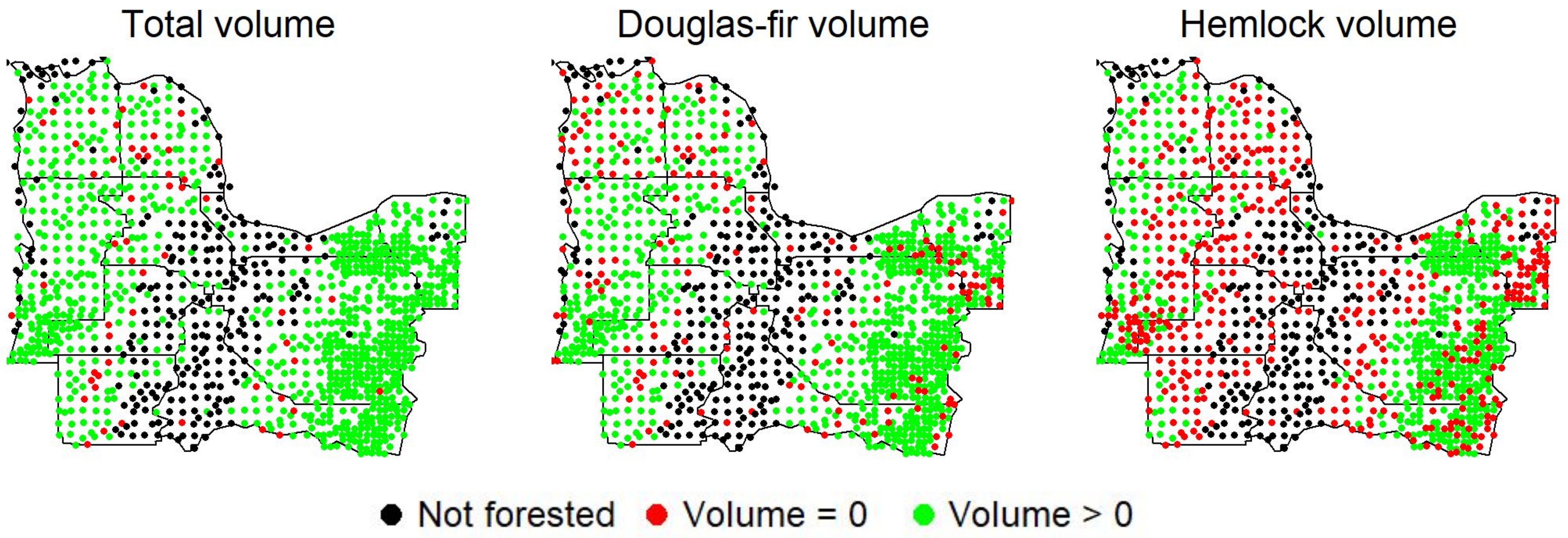
Modelling forest inventory variables is challenging

- Spatial correlation: locations close to each other “share” information
  - each plot does not represent a “full” unit of information.
  - the “shared” information can be used to improve prediction (i.e., kriging)
- Zero-inflated: a large proportion of the values of the variable are 0
  - Non-forest land, harvested areas, species not present…
  - The proportion of 0s increases as the domain becomes more restricted
- Often positive, very skewed
- This precludes using traditional modelling approaches based on the normal distribution

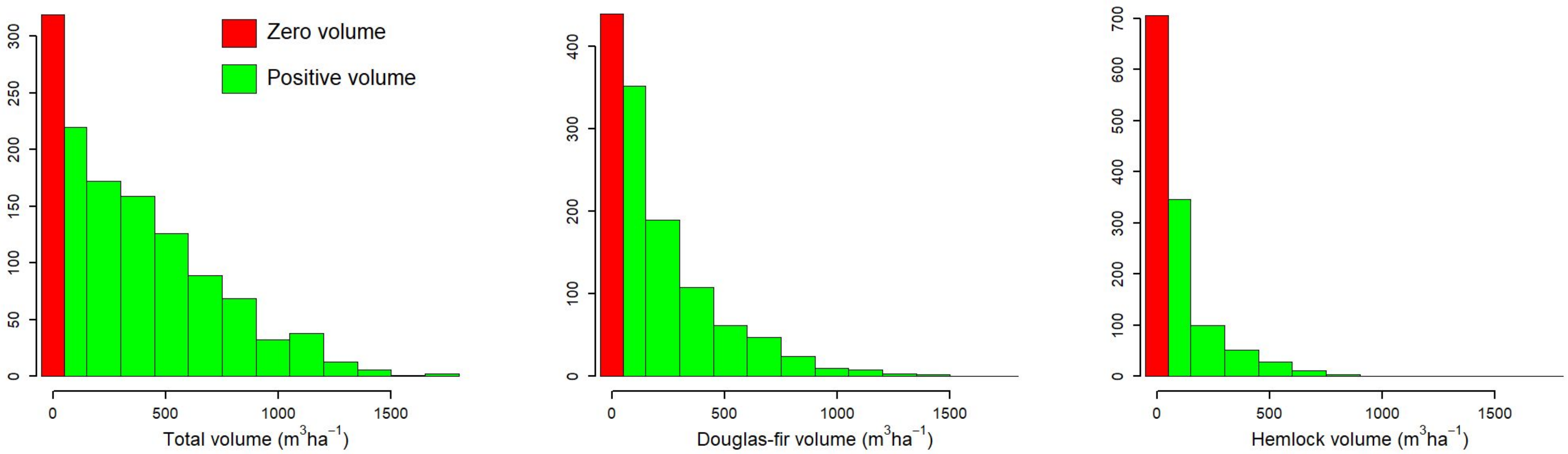
Example: timber volume in NW Oregon




Tillamook
Portland
Mt Hood
Salem



	Total	Douglas-fir	Hemlock
Percentage of plots with 0 volume	26	35	57

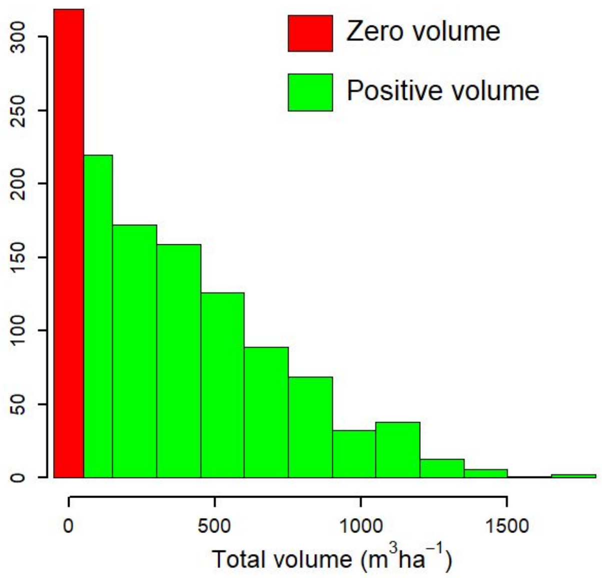


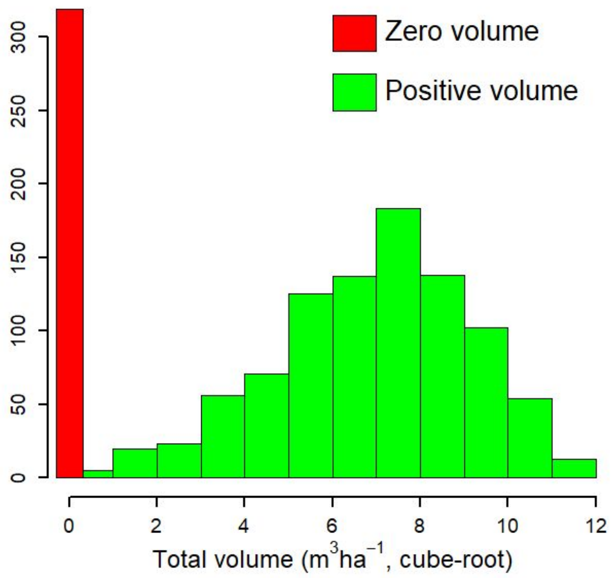
- Not only it is zero-inflated, but the distribution is highly skewed
- No simple transformation / solution can deal with those problems

What is a copula model?

- A copula is a multivariate distribution function for which the marginal probability distribution of each variable is uniform.
- Take advantage of the probability integral transformation
  - If V is a random variable with cumulative distribution function $F_V(v)$, then the variable $U = F_V(v)$ is uniformly distributed on $(0,1)$ [i.e., $P(U \leq u) = u$]
- The marginal distributions and the copula can be examined separately and fitted either separately or jointly using maximum likelihood.
- Main result (Sklar): every multivariate distribution function can be expressed in terms of its univariate marginal distributions and a copula describing the dependence among them.

Marginal distribution





- A cubic root transformation of the non-zero volumes worked best

Marginal distribution: zero-inflated gamma

- Zero-inflated gamma model to account for the excess 0s
- The observed volume (V, cube root) is a Bernoulli mixture of a 0 and a Gamma random variable:

$B \sim \mathrm{Bernoulli}(\pi)$ $\quad$ π: probability of volume > 0

$W \sim \mathrm{gamma}(\alpha, \beta)$ $\quad$ W: volume (cube root), given that it is not 0

$$V = (1 - B) \cdot 0 + B \cdot W$$

- Modelled the mean of B and W as a function of an indicator of forestland (based on of NLCD forest cover classes), Landsat tesseled cap "wetness" variable (tsc3), and their interaction

Gaussian copula: double transformation

- First transformation: estimate the cumulative distribution function of this marginal distribution, $U = F_V(v)$
- Second transformation: univariate standard normal, $\Phi^{-1}\left(F_V(v)\right)$ [Φ is the standard normal cumulative distribution function]
- Join together in a multivariate standard normal distribution
- Model the spatial dependence structure via a (rank) correlation matrix

$$C(v; \Sigma) = \Phi_\Sigma\left[\Phi^{-1}\left(F_1(v_1)\right), \dots, \Phi^{-1}\left(F_n(v_n)\right)\right]$$

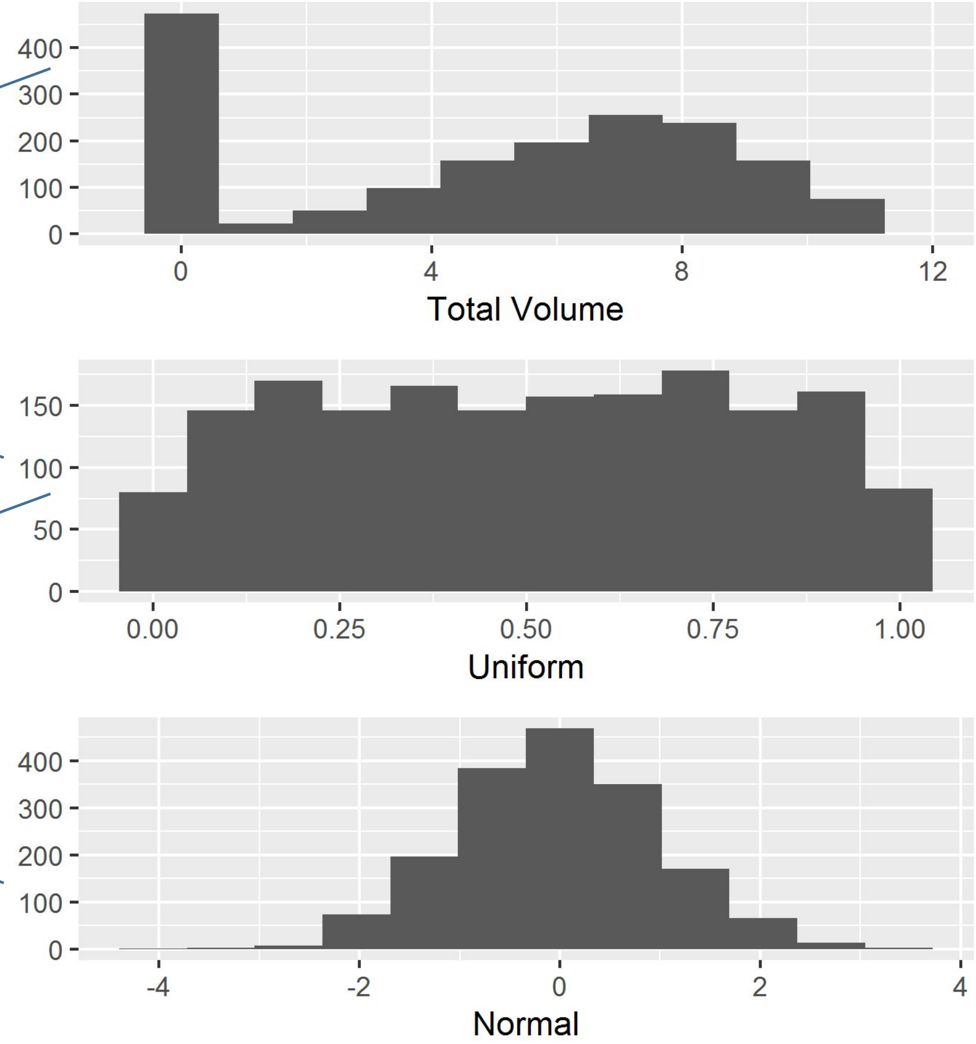
[Φ_Σ multivariate normal with mean 0 and correlation matrix Σ]

- Use the normal distribution tools for analysis /prediction (kriging)
- Reverse the transformations to the original scale

Transformation to Normal

$U = \mathrm{cdf}(V)$

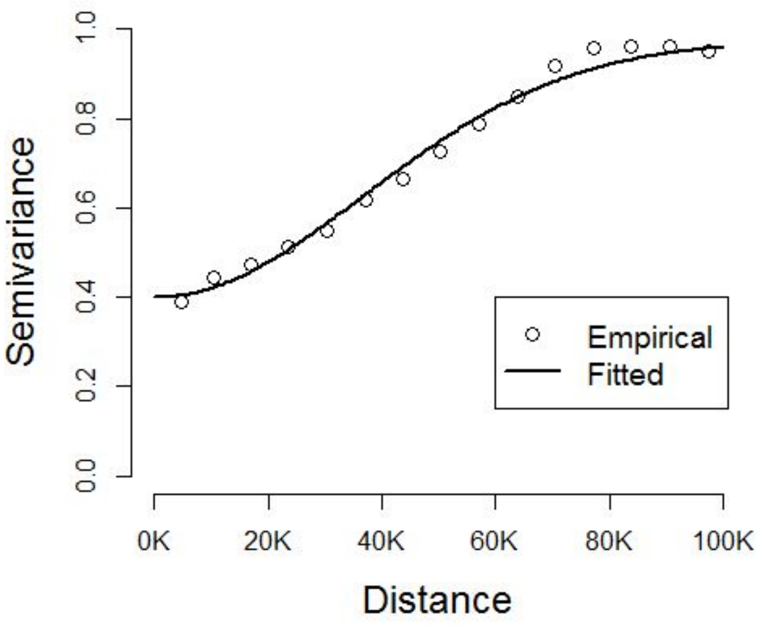
$Z = \text{inverse normal cdf}(U)$



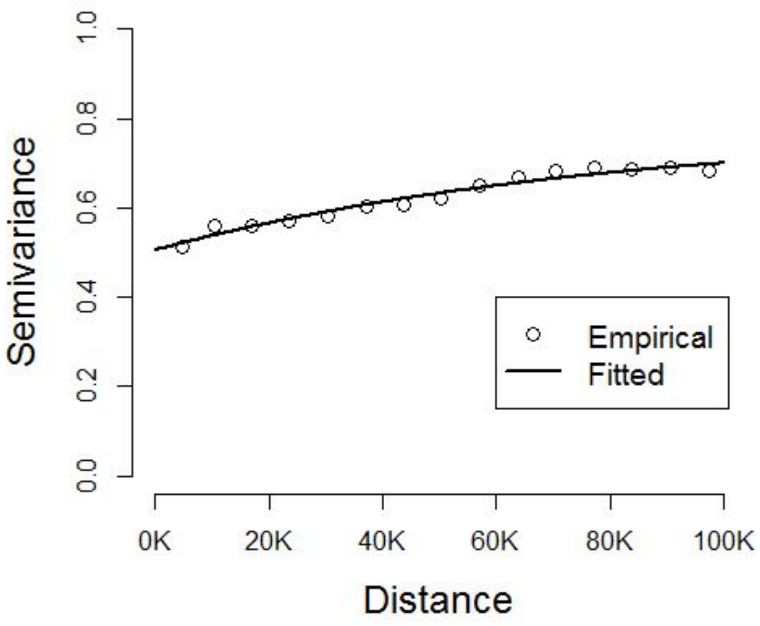
These steps are reversible.

Results

- For total volume, once we include the covariates, the spatial correlation becomes negligible



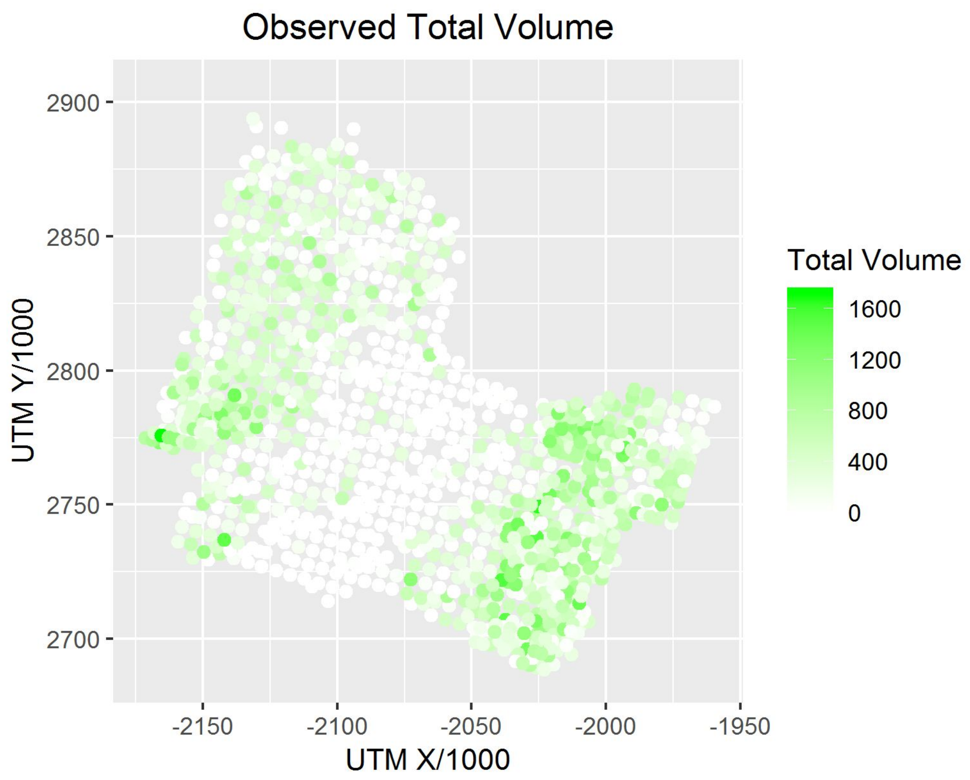
No-covariate model

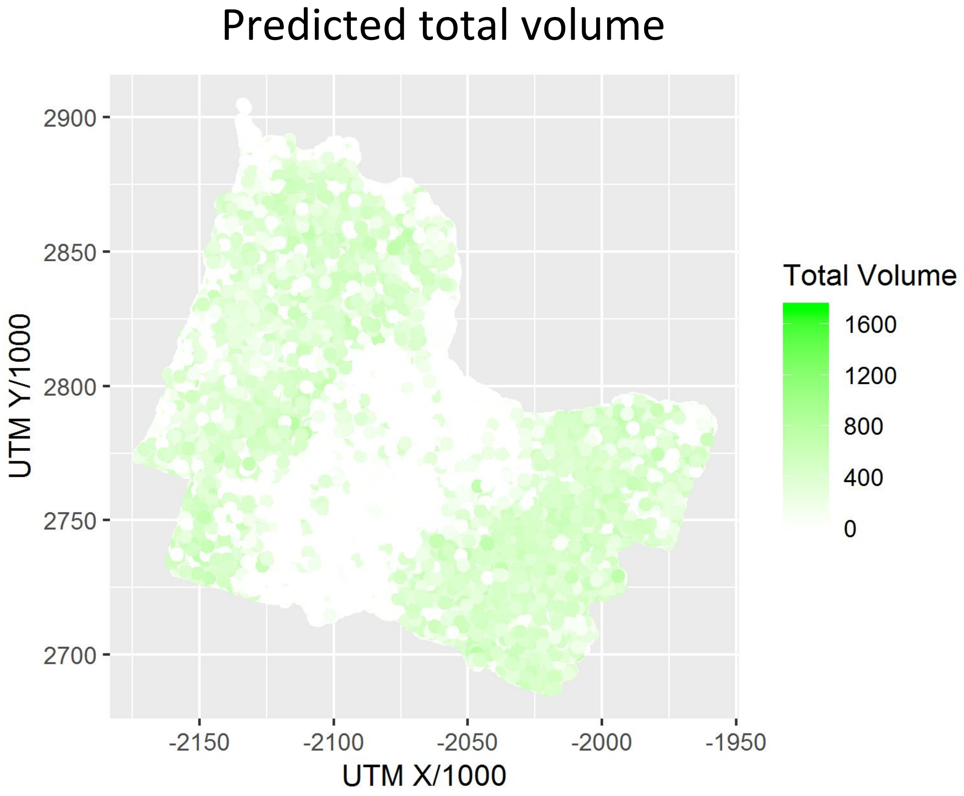


Modelled data

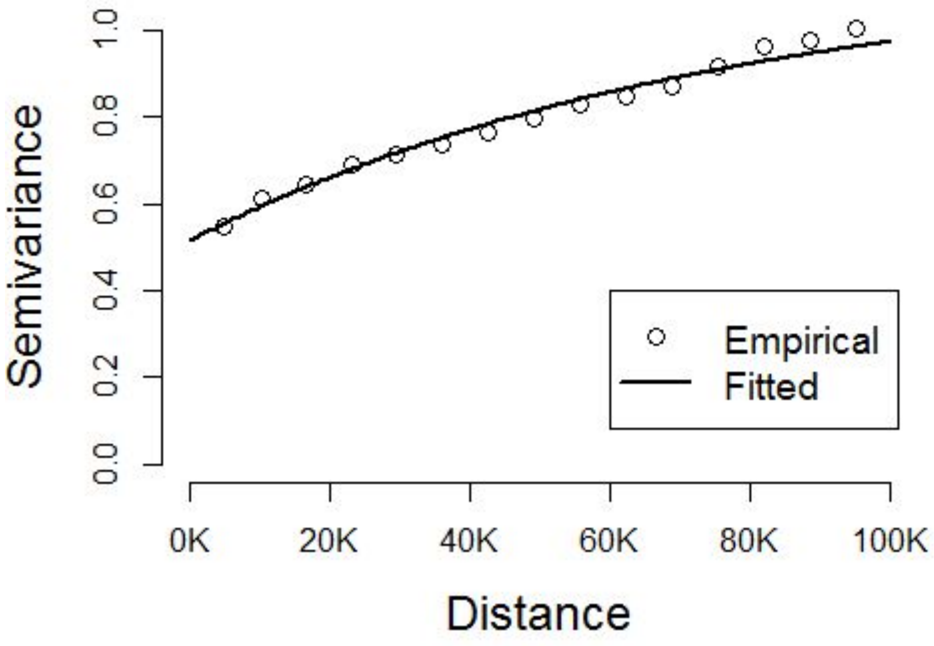
Total volume predictions

- Since spatial correlation is very weak, we can use a simple zero-inflated gamma model

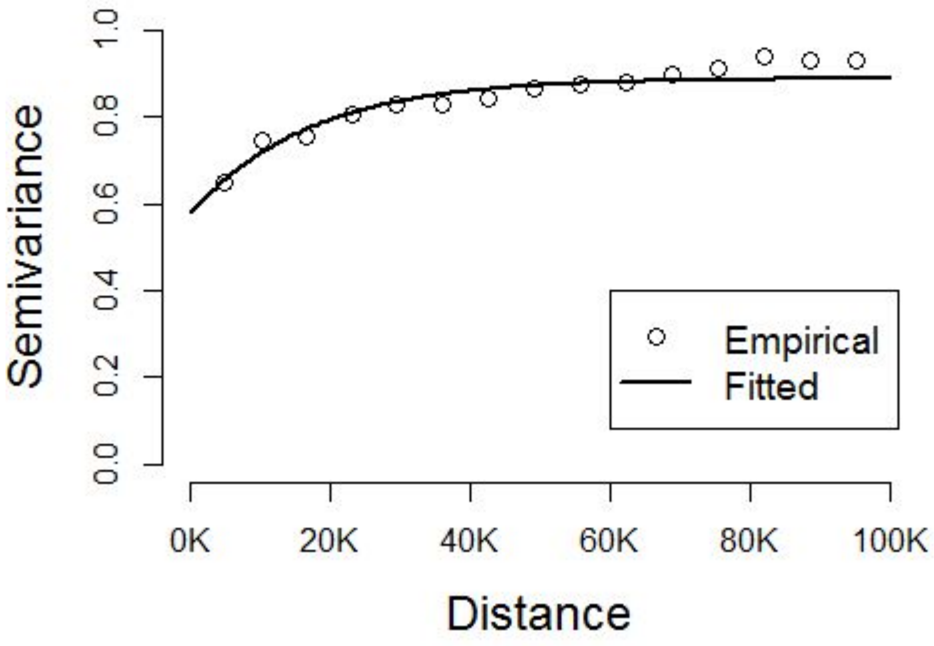




Hemlock volume - semivariograms

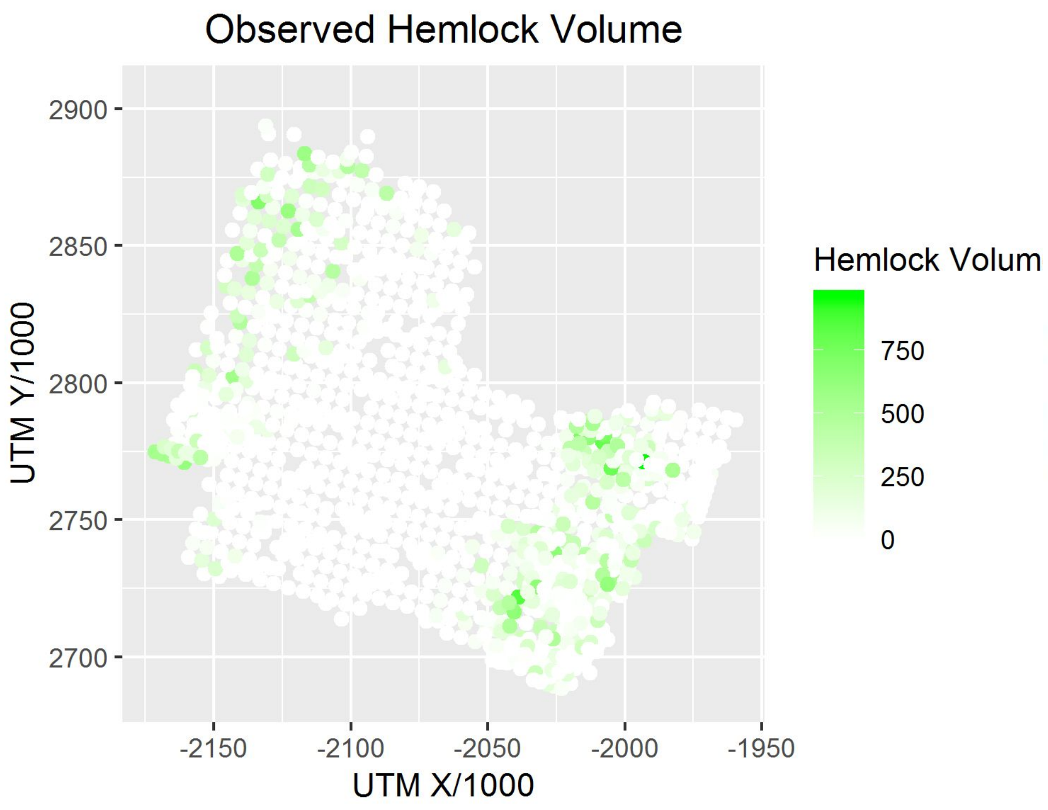


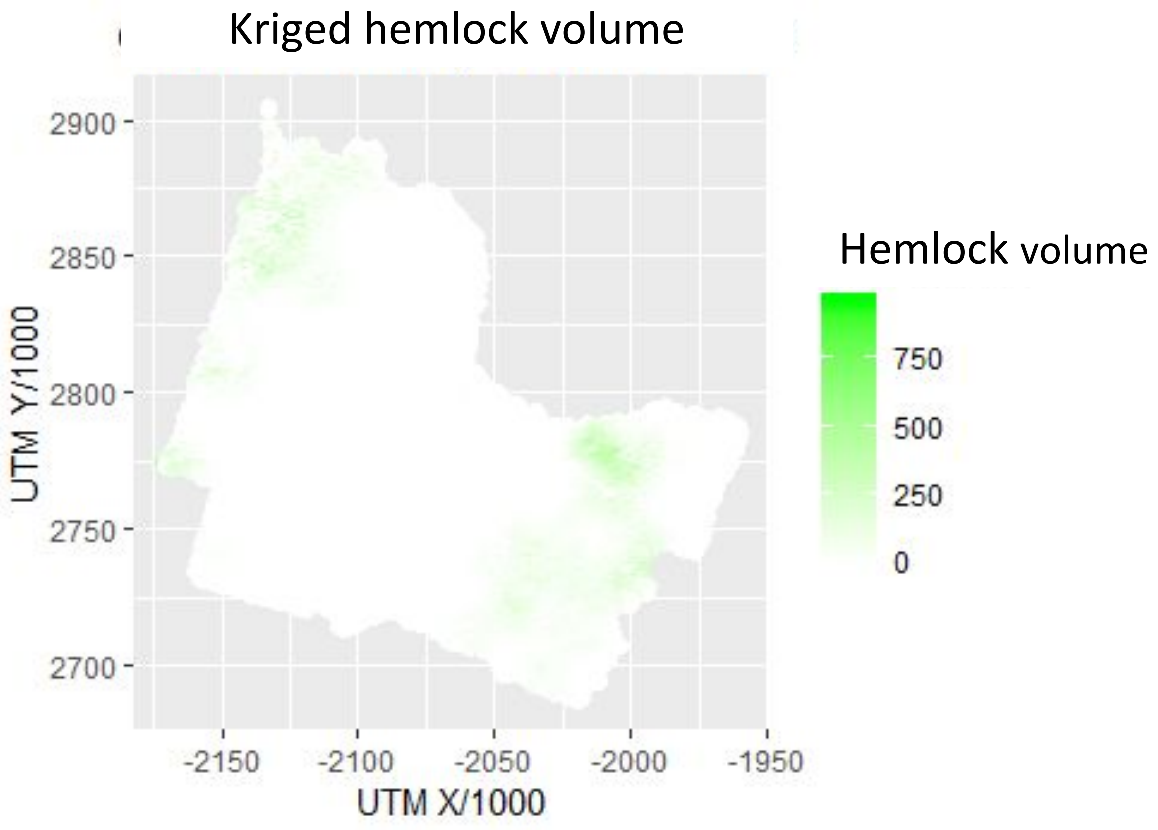
No-covariate model



Modelled data

Hemlock volume predictions





Conclusions and future work

- Gaussian copulas allow us to build realistic models for forest inventory variables, incorporating spatial correlation and non-standard distributions.
- Add covariates to build operational models
- Small area estimation: how to compute measures of uncertainty in the original scale
- High dimensional dataset, computational difficulties.