ST553 HW6

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Question 1

In this scenario we are given a one factor random effects model

$$y_{ij} = \mu + \alpha_i + \epsilon_{ij}$$

where $\epsilon \sim N(0, \sigma^2)$ and $\alpha_i \sim N(0, \sigma_\alpha^2)$. We also assume that the effects and errors are independent from one another.

(a)

What is the distribution of y_{ij}?

With the above assumptions, y_{ij} should be normally distributed around mean μ since $E[y_{ij}] = E[\mu + \alpha_i + \epsilon_{ij}] = \mu$.

The variance of y_{ij} can be found by looking at:

$$\begin{aligned} var[y_{ij}] &= var[\mu + \alpha_i + \epsilon_{ij}] \\ &= var[\alpha_i + \epsilon_{ij}] \\ &= var[\alpha_i] = var[\epsilon_{ij}] \\ &= \sigma_\alpha^2 + \sigma^2 \end{aligned}$$

Therefore we have $y_{ij} \sim N(\mu, \sigma_\alpha^2 + \sigma^2)$

(b)

Suppose that there are three levels of the random factor and two replicates at each level so that N = 6. Find the variance-covariance matrix of $Y = [y_{11}, y_{21}, y_{21}, y_{22}, y_{31}, y_{32}]$

It's pretty easy to derive the fact that if two observations come from the same level of the random factor, their covariance will be σ_α^2. If two different observations do not share the same random factor, their covariance will be 0.

	y_{11}	y_{12}	y_{21}	y_{22}	y_{31}	y_{32}
y_{11}	$\sigma^2 + \sigma_\alpha^2$	σ_α^2	0	0	0	0
y_{12}	σ_α^2	$\sigma^2 + \sigma_\alpha^2$	0	0	0	0
y_{21}	0	0	$\sigma^2 + \sigma_\alpha^2$	σ_α^2	0	0
y_{22}	0	0	σ_α^2	$\sigma^2 + \sigma_\alpha^2$	0	0
y_{31}	0	0	0	0	$\sigma^2 + \sigma_\alpha^2$	σ_α^2
y_{32}	0	0	0	0	σ_α^2	$\sigma^2 + \sigma_\alpha^2$

Problem 10.6

For this study, we want to examine the effects of two anticonvulsant drugs on the amount of neurotransmitter (GABA or γ-aminobutyric acid). In addition to these two drugs, we are also interested in determining whether the presence of calcium-binding proteins also affects the GABA production. This is a factorial design with three factors, one for each drug, and two levels apiece, present or absent.

Our primary qeustion of interest is *whether the drugs affect the release of this neurotransmitter and by how much.* We are also interested *if the protein-binding calcium calmodulin also changes the drugs affects.*

The overall omnibus F-test for our model returns the following ANOVA table:

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	1	1.09807500	1.09807500	9.75	0.0031
Error	46	5.18211667	0.11265471		
Corrected Total	47	6.28019167			

With a p-value of .0031, we have significant evidence to say that at least one of either the two drugs and the calmodulin have some effect on GABA production.

We can begin investigating the significant terms in the model by looking at the returned t-statistics in a full interaction model:

Source	DF	Type III SS	Mean Square	F Value	Pr > F
tri	1	0.92433750	0.92433750	29.70	<.0001
dia	1	3.06020417	3.06020417	98.32	<.0001
tri*dia	1	0.00633750	0.00633750	0.20	0.6542
cal	1	0.08520417	0.08520417	2.74	0.1058
tri*cal	1	0.23800417	0.23800417	7.65	0.0086
dia*cal	1	0.05133750	0.05133750	1.65	0.2064
tri*dia*cal	1	0.20720417	0.20720417	6.66	0.0137

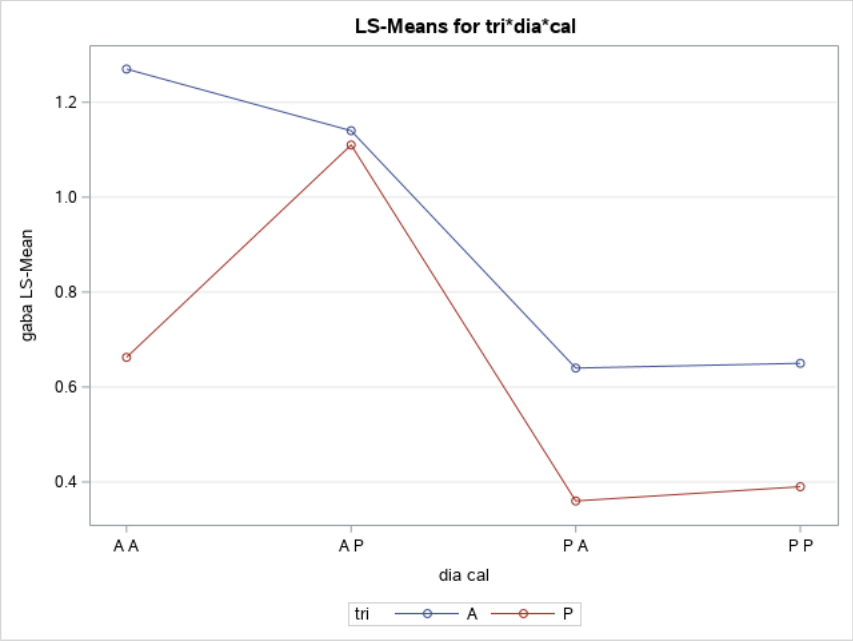
We see that both drugs trifluoperazine and diazepam are highly significant with pvalues $<.0001$. This suggests statistical evidence that these drugs *do affect* GABA production.

We also see that the interaction between trifluoperazine and calmodulin is significant with pvalue .0086 and the three way interaction between both drugs and calmodulin is significant with pvalue .0137. This indicates that we might want to investigate the interactions between the drugs and calmodulin.

We can get more specific numbers on the drug effects by investigating the contrasts comparing the absence and presence of the two drugs (trifluorperazine and diazepam):

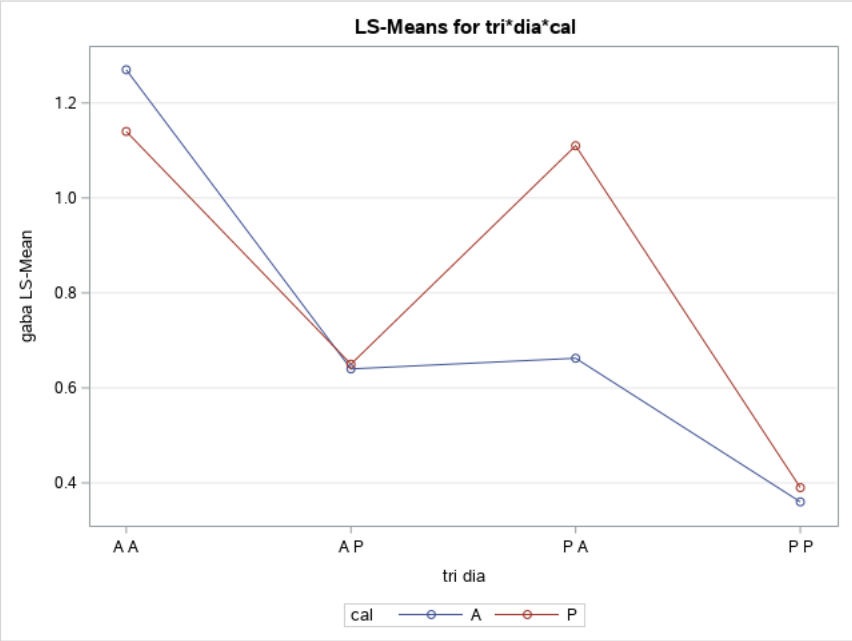
Parameter	Estimate	Standard Error	t Value	Pr > |t|	95% Confidence Limits	
triafluorperazine	0.29437500	0.05401714	5.45	<.0001	0.18520228	0.40354772
diazepam	0.53562500	0.05401714	9.92	<.0001	0.42645228	0.64479772
calmodulin	-0.08937500	0.05401714	-1.65	0.1058	-0.19854772	0.01979772

The first two contrasts in this table give us some important information about the effect of the drugs. With 95% confidence, the average amount of GABA neurotransmitter released is between .185 and .403 units lower for rats that received trifluoperazine more than rats who did not. Similarly, We can say that with 95% confidence, the average amount of GABA neurotransmitter released is between .426 and .644 units lower in rats that received diazepam than those that did not.



Examining this interaction plot split by trifluoperazine, we see that interestingly enough, when calmodulin is also present, the GABA production is actually much higher, particularly when diazepam is absent. Splitting this interaction plot by diazepam tells us similar information - there is an interaction effect between calmodulin and trifluoperazine that causes GABA production to be higher when the two are present in the same rat.

If we finally split this interaction plot by calmodulin, we can actually see further evidence that diazepam has a muting effect on GABA production - when it is present in the rat, GABA production is much lower than in rats where it is not present. This effect is particularly apparent in rats that are also given trifluoperazine. Rats that were given trifluoperazine and calmodulin have a much higher GABA production than rats that trifluoperazine, but that difference goes away and overall GABA production drops significantly when diazepam is present. This change suggests why there is statistically significant three-way interaction effect between diazepam, trifluoperazine, and calmodulin: *diazepam appears to negate the effect of calmodulin in rats that also have trifluoperazine.*


LS-Means for tri*dia*cal
gaba LS-Mean
1.2
1.0
0.8
0.6
0.4
A A
A P
P A
P P
tri dia
cal
A
P

Appendix

Here is the SAS Code for Problem 10.6

```
filename drugs 'pr10.6.csv';
data dat;
    infile drugs firstobs=2 dlm=',';
    input tri $ dia $ cal $ gaba;
run;
proc print data=dat;
run;

proc glm data=dat plots=diagnostics;
    class tri dia cal;
    model gaba=tri|dia|cal /clparm;
    estimates 'triafluorperazine' tri 1 -1;
    estimates 'diazepam' dia 1 -1;
    estimates 'calmodulin' cal 1 -1;
    lsmeans tri*dia*cal /plots=meanplot(join) slice=tri;
    lsmeans tri*dia*cal /plots=meanplot(join) slice=dia;
    lsmeans tri*dia*cal /plots=meanplot(join) slice=cal;
    lsmeans tri*dia*cal/adjust=tukey cl;
run;
```